Slide-Together Structures

Rinus Roelofs
Sculptor
Lansinkweg 28
7553AL Hengelo
the Netherlands
E-mail: rinusroelofs@hetnet.nl

Abstract

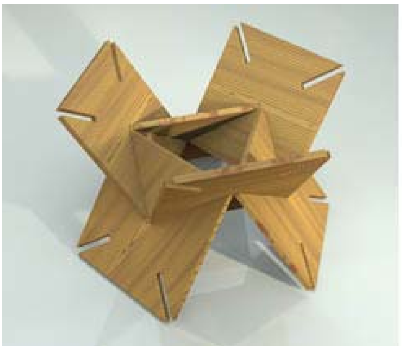
About ten years ago I discovered an interesting way to construct a tetrahedral shape by sliding together four rectangular planes in a certain way. By using halfway cuts in the planes it was possible to slide them together, all at once, to become the enclosed tetrahedron. This way of constructing objects and structures, finite and infinite, has been of my interest from then on. In this paper I will give an insight into some of the results of my research in this field. Besides halfway cuts I examined some other ways of slide-together structures.

1. Introduction

1.1. "Slide-together". At Bridges 2004 George Hart presented his "slide-togethers": polyhedral constructions built with simple flat paper elements, which were slid together [1]. Each "slide-together" was made from identical copies of a single type of regular polygon with slits cut at the proper locations. The pieces in these constructions had to be bent during the assembly. In my search I tried to focus on structures that can be built with rigid elements. So in most cases sliding the pieces together to form the final structure is possible without bending the pieces.

Figure 1 a,b,c: *Basic principle - halfway cut.*

1.2. Halfway cuts. With the use of halfway cuts as in Figure 1 we can combine several pieces to construct complex structures by just sliding them together. The simplest and most direct way to do this is by adding the pieces one by one. However, when more than one slide direction is used this is not always possible. And in some occasions it is even necessary to put the pieces together all at the same time. In Figure 2 we see a construction built from six identical square pieces with four halfway cuts each. When making the assembly we could start with one piece and then add the other pieces one by one. Doing so you will notice that you will have a problem when you want to add the last piece. It is better to make groups of three pieces first and then slide together the two groups. In the completed structure you can distinguish three different slide directions. We can use any of these directions to split the structure into two parts. And this is also the case in the more complex structure of Figure 3, which is an extension of Figure 2.



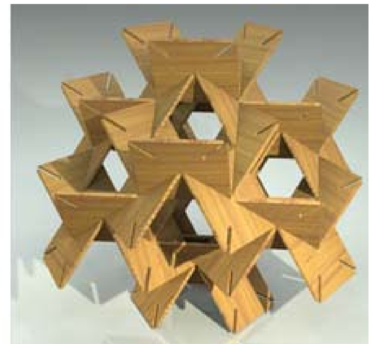


Figure 2a, 2b: *Ring formed by 6 square pieces.* **Figure 3**: *3D structure*

1.3. Four slide directions. In Figure 4 we see an assembly of four rectangular shapes with two halfway cuts each. The enclosed shape is a tetrahedron. The only way to put the pieces together is sliding them together all at the same time. The discovery of this structure was the start for my research. Many questions arose. For instance: are there more, similar, structures like this one? How can we make infinite slide-together structures? Is it possible to use even more slide directions? And which systems can be assembled only by sliding together all the pieces at the same time?

Besides the extension of the basic tetrahedral slide construction (Figure 5) I also found another way to build an 'infinite' tetrahedral structure as can be seen in Figure 6. This sculpture consists out of 104 flat triangles, slid together all at the same time.

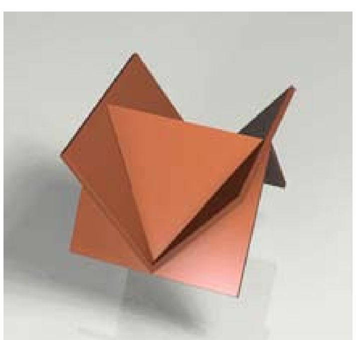
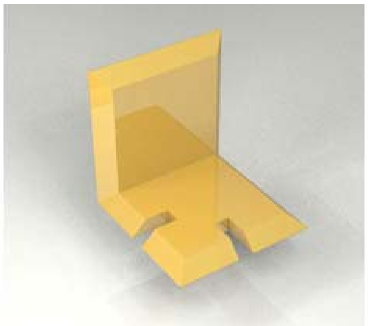


Figure 4, 5, 6: *Tetrahedral structures.*

2. 'Folded' Elements

2.1. Double pieces. The two infinite structures in Figure 3 and Figure 5 are both generated by doubling the basic pieces. A first idea in trying to find new shapes and structures was by introducing a 'folding' line into the pieces. Not as a hinge between the two 'half' pieces but as a rigid connection between the parts. In the first example with the basic structure of the ring of Figure 2 (three slide directions) the use of the 'folded' pieces leads to an object that looks like 2 connected cubes. And also this object can only be taken apart in two groups of three pieces before you can separate all the pieces from each other.



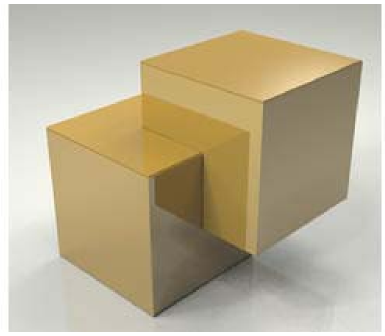
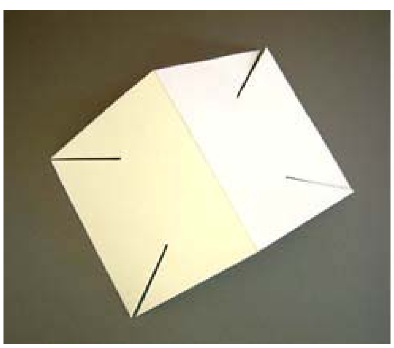


Figure 7, 8: *Two connected cubes.*

2.2. Slide together cylinders. When we double the pieces of the tetrahedral structure, we can create a variety of different cylinders by changing the angle between the 'halves'. All these cylinders can only be taken apart by sliding away all the pieces at the same time. The cylinders can be seen as a stack of antiprisms. The first cylinder in the set is a stack of tetrahedra, which also can be seen as the first member of antiprisms. The next member, based on the three-sided antiprism, has the same structure as the ring of Figure 2, and therefore you might expect that it can be split into two parts by sliding along one of the sliding lines, but because of the doubling of the pieces the number of sliding lines has also been doubled, which changes the sliding system.



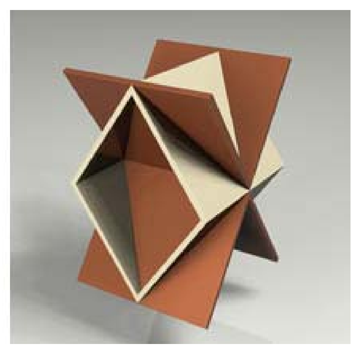


Figure 9, 10, 11: *Elements.*

Figure 12, 13: *Cylinders - Paper models*

. **Figure 14**: *Cylinder - Steel*

3. Flat Infinite Structures with Folded Elements

3.1. 2D infinite. The cylindrical structure is infinite in one direction. A cylinder can be cut and unrolled to become a 2D flat surface. In Figures 15, 16, 17 you can see how this step leads towards a 2D infinite slide together structure. From the two types of elements, which were used to build the cylinder, only one is left in the flat structure. Again you see the enclosed tetrahedra, which means that also in this structure four sliding directions are used. So we can conclude that these elements will have to slide together all at the same time too.

Figure 15,16,17: *Unrolling the cylinder.*

3.2. Variations. There are a few interesting variations of this type of slide-together structures. When we make the pieces a little bit more complicated by adding extra parts as in Figure 18 we will get nice and strong double layered structures (Figure 19 and 20).

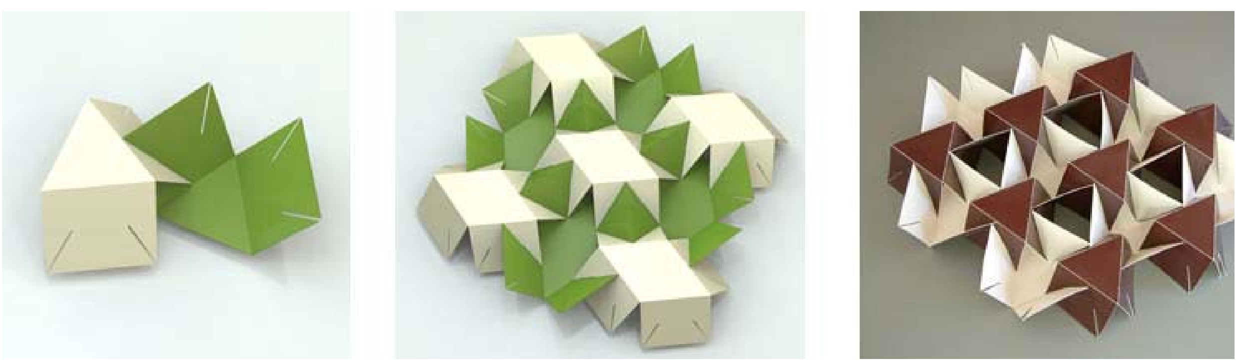


Figure 18,19,20: *Double layer.*

In the case of Figures 21 and 22 a double connection, a parallel pair of sliding lines, between pairs of pieces is used. Special about these two examples is that you first will have to make rows, and then slide the structure together row by row.



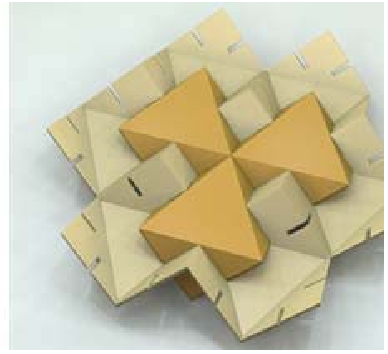


Figure 21a,b,c: *Double connection.*

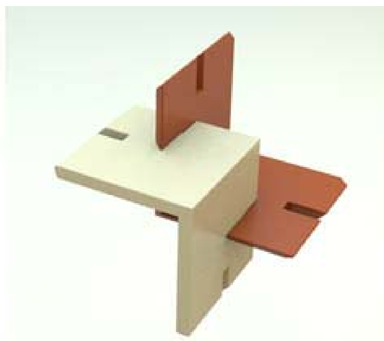
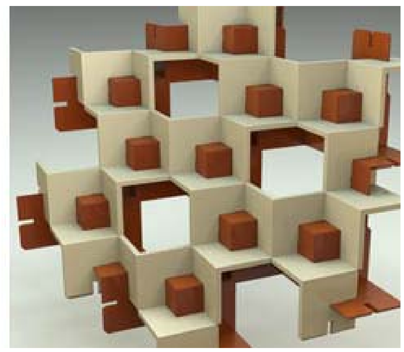




Figure 22a,b,c: *Double connection.*

4. "Tile Rotation"

4.1. Bending the pieces. There is one special group 2D slide-together structures that can only be assembled when we allow bending the pieces during the assembly. I decided to add this group because of a very interesting property. The pieces of this group consist of two connected triangles with both two halfway cuts (Figure 23) and in the final structure the double connection is used (Figure 24). Because now the pair of slide lines is not a parallel pair, bending of the pieces is needed. When we look at the final structure we can recognize a tiling pattern lying on the surface: a pattern with small and big square tiles (Figure 25). And when we look through the structure we can recognize the same tiling but now with the opposite orientation: left turning instead of right turning.

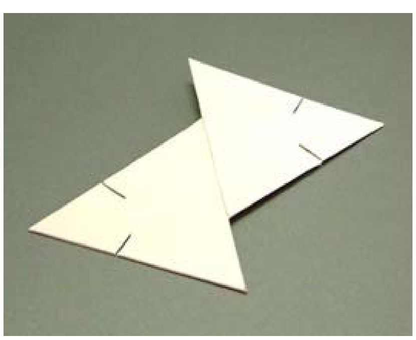
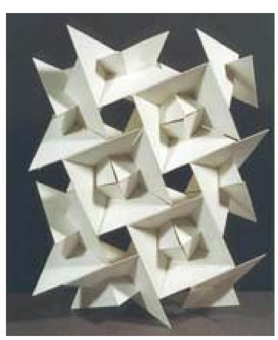
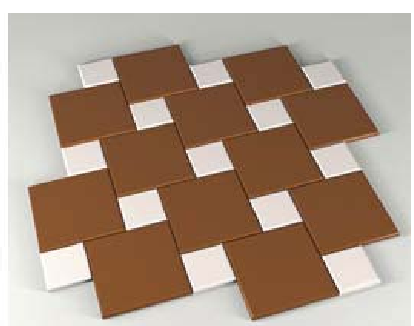




Figure 23,24,25: *Tile rotation 4,4.*

When we project the upper tiling onto the lower tiling we see that the center of each tile is in the same position and each tile seems to have been rotated around its own center. I found a few other tilings to which I could apply this operation, which I called 'tile rotation'. I wanted to examine whether it was possible to construct a slide-together structure from tilings with this property. In the tiling of Figure 26 we can change the orientation by rotating the hexagons and the triangles, each around their own center. And so from this tiling I was able to design the accompanying slide-together structure (Figure 27 and 28).

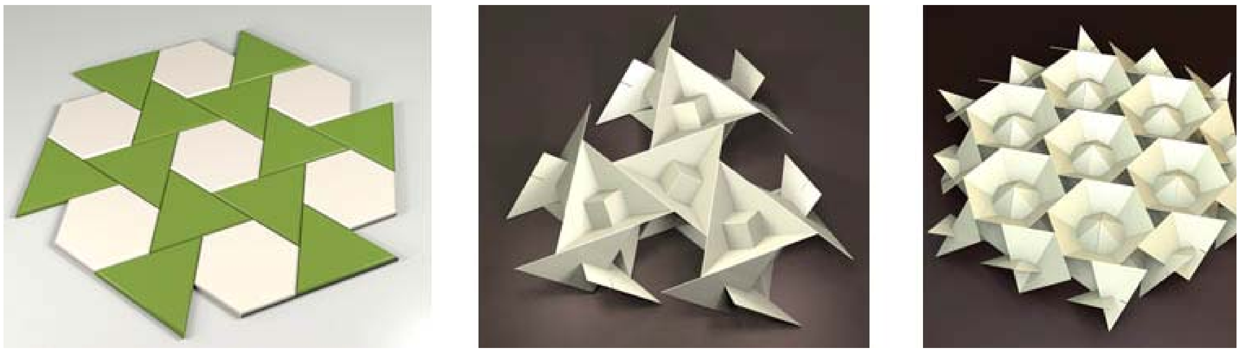


Figure 26,27,28: *Tile rotation 6,3.*

5. Mortise-and-tenon Joints

5.1. Introduction. Another way of making slide-together structures is the use of mortise-and-tenon joints. The structure in Figure 29 consists of six equal elements and has some similarities with the ring structure of Figure 2. Here too we have got two possibilities. You either take it apart into two groups first or you take it apart by moving all the elements away from the center at the same time. And to become an infinite structure we can again double the elements. See Figure 30.

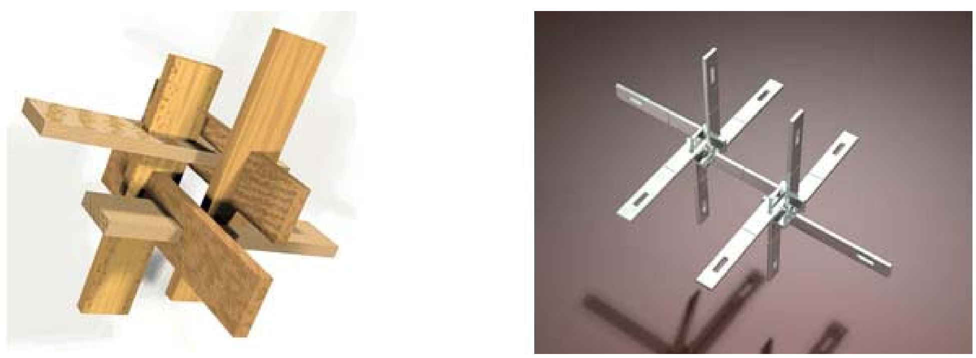


Figure 29,30: *Mortise-and-tenon joints.*

5.2. One mortise two tenons. The idea of using two tenons instead of one to fill one mortise opened up a completely new field of slide together structures. In many 3d structures you can find intersecting planes. The idea was, when you have a set of intersecting planes, to split up one of the planes in such a way that you get two tenons crossing the intersection line with the other plane from opposite directions. An example of such a tiling of a plane can be seen in Figure 31. Here the plane is divided into three equal pieces. The division line has a Z-shape to create the tenons. The Z-cut is made in such a way that the pieces can slide apart when moving from the center. Now each piece (Figure 32) has two sides with a tenon and in the middle is a mortise of such a size that two such tenons, coming from different directions, fit in precisely. Sliding together 12 pieces creates an intersection of four planes (Figure 33). The shape of the completed construction can be recognized as the cuboctahedron. Sliding together the 12 pieces is fairly easy to do. For me it was a surprise that taking the cuboctahedron apart is practically impossible.

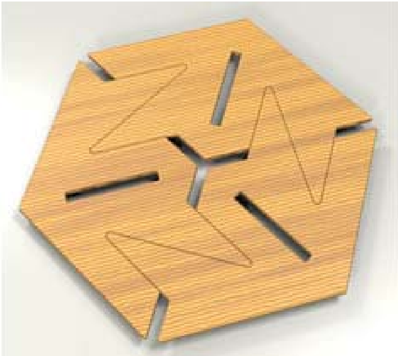
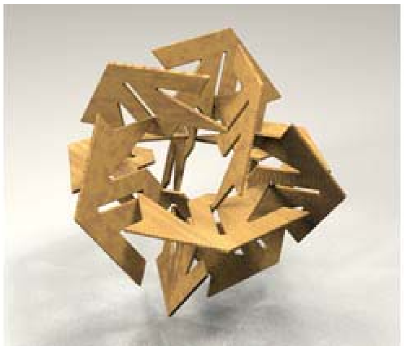


Figure 31,32: *Pieces.*



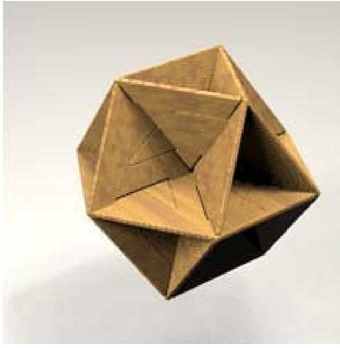


Figure 33a,b,c: *Cuboctahedron.*

5.3. Polyhedra. The cuboctahedron is one of the Archimedean polyhedra in which you can recognize a set of intersecting planes. The four planes have one intersection point in the center of the polyhedron; each of these planes is divided into three pieces. And now all the pieces can be moved towards this center point and the tenons will slide smoothly into the mortises.

Similar situations can be found in a few other polyhedra: the octahedron with three intersecting planes and the icosidodecahedron with six intersecting planes. In Figure 34 you can see a subdivision of one of the six planes of the icosidodecahedron into five equal pieces. Thirty such pieces can be slid together to create the icosidodecahedron of Figure 36.

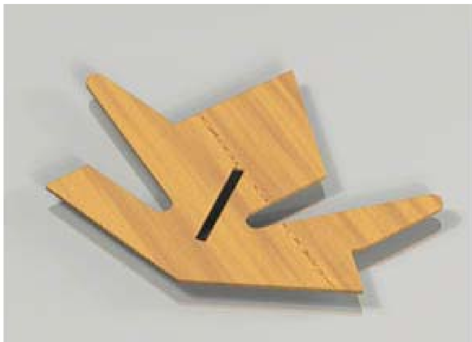


Figure 34,35: *Pieces.*

Figure 36: *Icosidodecahedron.*

5.4. General intersections. So far all the slide together structures with mortise-and-tenon joints were regular, all the pieces in the construction were equal. But the Z-cut method can be applied also in other cases. As an example a thirty piece cylindrical structure is shown in Figure 38. Five different types of pieces were used to make this object.

Figure 37,38a,38b: *Cylinder.*

6. Further Generalization

6.1. Parallel Faces. In section 5 in each structure any two planes had an intersection line. There were no parallel faces. The number of pieces then can be calculated by multiplying the number of faces n by (n-1). It is however no condition that each plane should intersect every other. Also in situations where we have parallel faces we can use the Z-cut method to construct a set of pieces for a slide-together structure. The cubical construction in Figure 39 is made of twenty four equal pieces. Each of the six faces is divided into four parts using the Z-cut, now leaving a square hole in the middle.

Figure 39: *Cubic.*

6.2. Outside the center. Looking at the construction of Figure 39 we can say that the connection between the pieces takes place at the vertices of a cube and that pairs of pieces follow the edges of the cube. Describing the structure in this way gives us a method of using the same steps starting with other polyhedra, for instance the tetrahedron or the dodecahedron. Also in these polyhedra three faces meet at every vertex, so the same translation can be used. In case of the dodecahedron (Figure 41) we end up with a 60-piece slide-together structure.

Figure 40a,b,c: *Tetrahedron.*

Figure 41a,b: *Dodecahedron.*

6.3. Parallel tenons. Instead of opposite tenons we can also use parallel tenons. In this way it is possible to create box like slide together structures as you can see in the examples (figure 42 and 43).

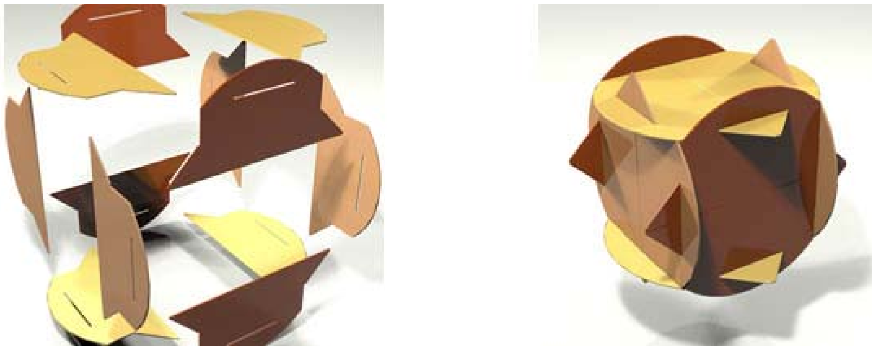


Figure 42a, 42b: *Cube.*

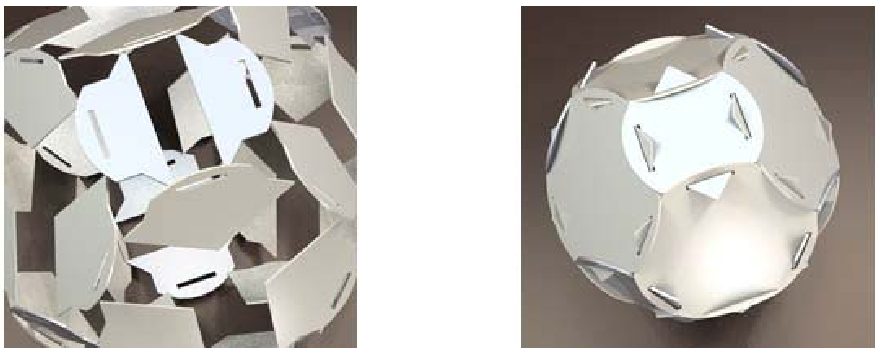


Figure 43a,b: *Truncated Octahedron.*

7. Coxeter Polyhedra

7.1. Infinite structures. The question arose how to create infinite structures using the kind of pieces developed in the previous section. Staying in the field of polyhedra we can then take the infinite regular polyhedra, found by H.S.M. Coxeter [2] as the basic structures. Both methods, tenons in opposite directions, and parallel tenons can be applied to make slide together structures. It gives the idea of an implosion seeing all the pieces coming together to form the slide together structure.

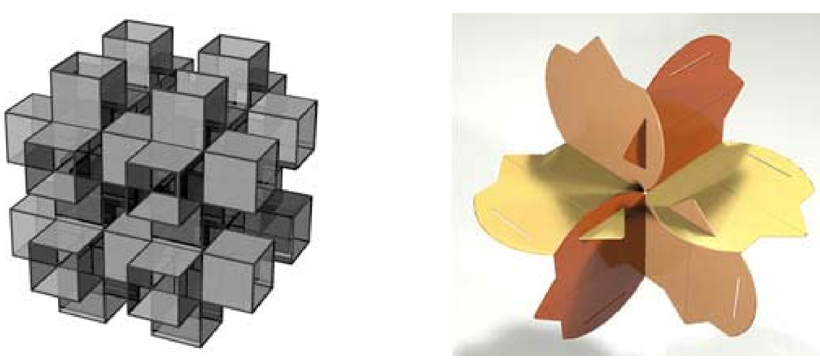


Figure 44,45: *A Coxeter glide together structure.*

8. Further steps

8.1. Alternate shapes. To open up the field for further research I experimented with some other shapes. The first group is a special set of star shaped polyhedra. The objects are constructed by applying a certain transformation to the octahedron, the cube and the dodecahedron. All three shapes can be assembled with a kind of pyramid shape. In each object the separate pieces will slide towards the center at the same time.

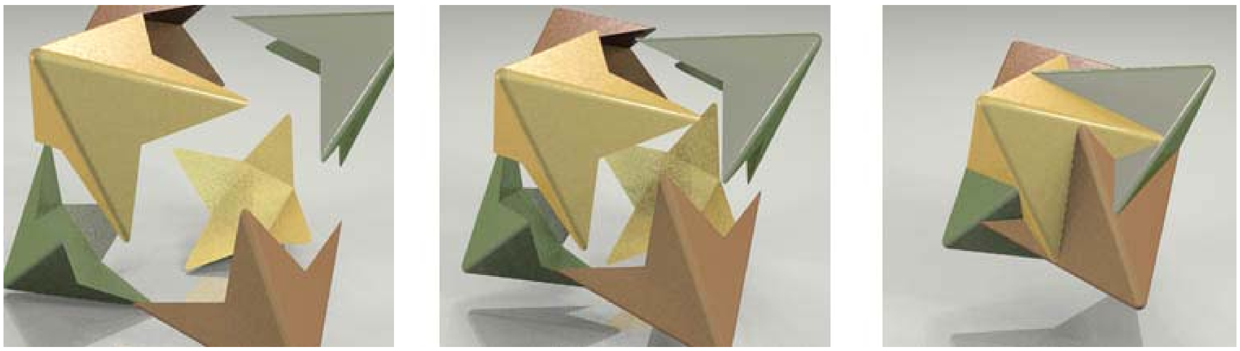


Figure 46: *A octahedron.*

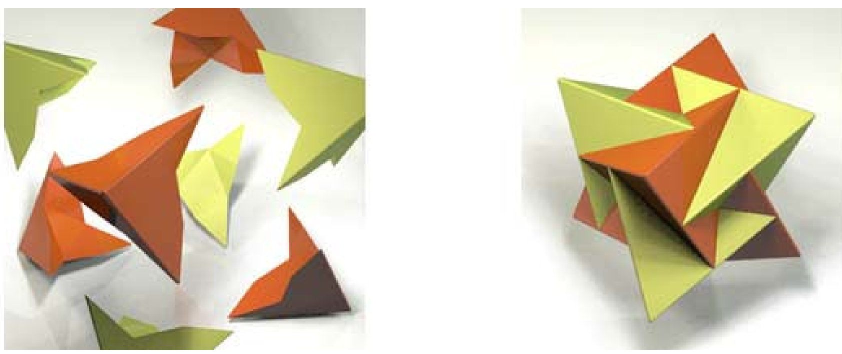


Figure 47: *A cube.*

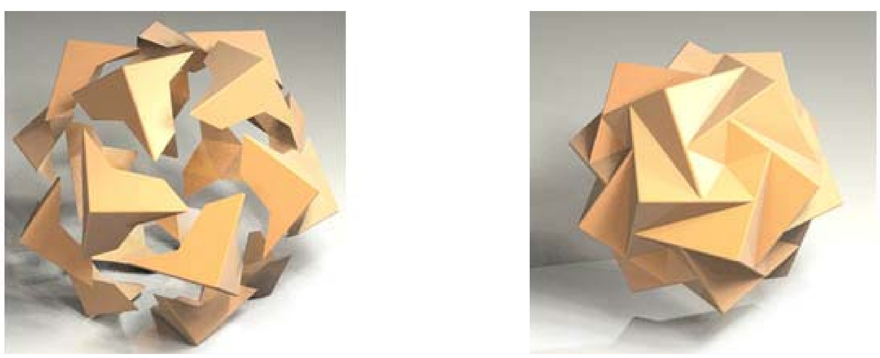


Figure 48: *A dodecahedron.*

8.2. Escher's star - Rhombic star. Besides other shapes of the pieces also other types of movements can be studied to find new possibilities. The 12-pointed star that M.C. Escher uses in his print "Gravitation" [3] can be made from 12 flat pentagonal shapes. The question is whether it is possible to slide the pieces together to become the complete star. I decided to study the rhombic version of the 12-pointed star and found out that it was even possible to create an object with smaller holes then M.C. Escher used. The Rhombic star can be made as a slide together structure. But now we have to slide the pieces together not only by translation but we also have to rotate the pieces during the translation. It is a nice spiraling movement, which brings all the pieces smoothly together to a 12-pointed rhombic star. And in a way this brings us back to the beginning where as apart from the moving aspect there is a clear connection between this object and some of George Hart's sculptures [4].

Figure 49: *M.C. Escher's 12-pointed star.*

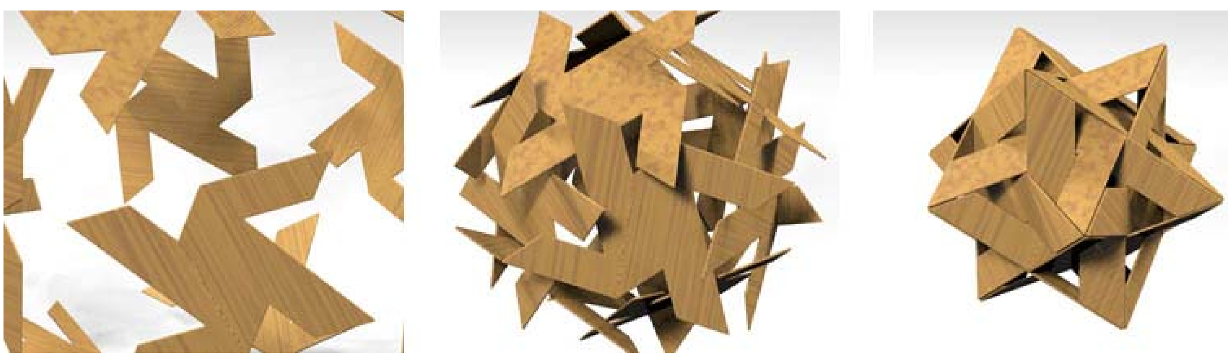


Figure 50a,b,c: *The 12-pointed rhombic star.*

References

[1] George W. Hart, *"Slide-Together" Geometric Paper Constructions,* Workshop at Bridges 2004, http://www.georgehart.com

[2] H.S.M. Coxeter, *Twelve geometric Essays,* Southern Illinois University Press 1968

[3] M.C. Escher, *Grafiek en Tekeningen,* van Tijl 1972

[4] George W. Hart, *Sculpture from Symmetrically Arranged Planar Components,* ISAMA-Bridges 2003